Environment and Development Economics 8: 219–246 © 2003 Cambridge University Press
DOI:10.1017/S1355770X03000123 Printed in the United Kingdom

Theory and Applications

Economic development, demographics, and renewable resources: a dynamical systems approach

JOHN M. ANDERIES
School of Life Sciences and Center for Environmental Studies, Arizona State University, Tempe, AZ 85287. E-mail: m.anderies@asu.edu

ABSTRACT. Many developing economies are faced with growing populations and deteriorating natural resources. It is often argued that economic growth will arrest these trends via cleaner technology and social change accompanied by reduced birth rates. Two questions arise: (1) What general economic, demographic, and ecological conditions favor this scenario? and (2) What adjustments, technological, demographic, or ecological, are more important in realizing this scenario? I address these questions using a two-sector growth model which includes human demographics and a renewable resource base. Using powerful numerical bifurcation techniques and rescaling arguments, I obtain the following general results. If the regeneration rate of the renewable resource base is slow relative to the rate of economic growth, population overshoot and resource collapse is more likely. Demographic adjustments are more important than technological adjustments in avoiding renewable resource degradation. Several related results are presented that support these general findings.

1. Introduction

The interaction between human societies and the resource bases upon which they depend can be characterized by the dynamic tension between three interacting elements: (1) human population dynamics, (2) natural resources (e.g. productive land), and (3) technological progress and economic growth. Scholars have long been interested in the interactions between these elements. Because of the complexity of these interactions, studies have tended to focus on two of the three elements. Namely, there are studies on population and natural resources, e.g. Boserup (1965), Malthus (1999); economic growth, resources and the environment, e.g. Solow (1974), Stiglitz (1974), Dasgupta and Heal (1974), Cass and Mitra (1991); and economic growth and population dynamics, e.g. Leibenstein

I would like to thank J. Brander, W. Brock, C. Perrings, M. Janssen, N. Abel and three anonymous reviewers for helpful comments and advice on earlier drafts of the manuscript.

(1954), Galor and Weil (2000), and Barro and Sala-i-Martin (1995); but very few that attempt to address all three.

These studies all address aspects of a core question: What are the implications of population growth and technological change in the face of a degradable resource base for the welfare of future generations? Complex feedbacks between population, growth, technology, and natural resources make this question impossible to answer. For example, increasing population increases the pressure on the resource base. The resulting resource degradation may decrease *per capita* food output which reduces the rate of population growth. This feedback loop, first proposed by Malthus in 1798, causes the population to self equilibrate, i.e. resource availability drives population dynamics. Boserup (1965), on the other hand, suggested that the feedback works in the other direction. Namely, increased population pressure stimulates technological change which, in turn, increases *per capita* food output. This relieves checks on the population allowing it to continually grow while the resource base is exploited more and more intensively.

Boserup indicates that as population pressure increases, society moves through a sequence of cultivation techniques: forest–fallow cultivation → bush–fallow cultivation → short-fallow cultivation → annual cropping → multicropping. She suggests that, as this progression occurs, output per unit of labor input decreases. More generally, in order to increase agricultural output per unit area, energy and human-made capital must be substituted at an ever-increasing rate for natural capital. Are there limits to this process?

There has been a long association between the formal neoclassical theory of economic growth and the study of this process of substitution of human-made capital for natural capital. The most recent wave of work in this area probably began with the well-known work of Solow (1974) who used growth models to study intergenerational equity with exhaustible resources, and papers by Stiglitz (1974), and Dasgupta and Heal (1974) which address the optimal depletion of exhaustible resources. This work has been extended in many directions including studying the feasibility of maintaining consumption indefinitely (Cass and Mitra, 1991) and optimal investment rules for doing so (Hartwick, 1977).

These models have led to research on how 'sustainability' can be defined (Pezzey, 1992; Toman, Pezzey, and Krautkraemer, 1994; Pezzey, 1997). Growth models have been extended to incorporate renewable resources, assimilative capacity, pollution, and defensive expenditures (see Beltratti, 1997, for an excellent summary). The main thrust of this work is to establish conditions for which (optimal) sustained economic development is possible in the face of an environmental constraint.

Even if physical checks on population growth can be perpetually relieved, human populations cannot, logically, grow indefinitely. For the short timescales implicit in growth models, such checks are not significant. Over timescales of the order of several hundred years, they become important. Specifically, as economic growth proceeds, individuals making economically rational fertility decisions reduce birth rates and check population growth endogenously. Caldwell (1976) argues that the key driver behind this change is a reversal in the direction of intergenerational wealth flows from children to parents. Such fertility choices have been included in standard growth models, e.g. Barro and Sala-i-Martin (1995), and some interesting extensions thereof. Galor and Weil (2000) have developed a complex growth model in which agents make child quality/quantity trade-offs in their fertility choice with a minimum consumption constraint. They illustrate the transition from a 'Malthusian Regime' where the relationship between population growth rate and output is positive, to a 'Modern Growth Regime' where this relationship is negative. They do not consider resources or the environment.

The shift from a positive relationship between output and population growth to a negative relationship is essential in avoiding overshoot and collapse type behavior like that illustrated in the recent model of Easter Island by Brander and Taylor (1998). In this 'Malthusian' model, there is no economic growth or mechanism for a demographic transition as in the Galor and Weil model. The overshoot and collapse phenomenon is driven by the underlying resource dynamics. In a modern development context, whether or not a population overshoots its resource base will depend on how the dynamic tension between the environment, economic growth, and demographic patterns play out. Overshoot and collapse occurs when the Boserupian feedback loop of population growth leading to innovation leading to additional population growth spins out of control and eventually destroys the resource base. The hope is that this process will be checked by countervailing forces generated by growth and technological change.

Richard Baldwin (1994) suggests two main checks that will prevent the Boserupian feedback loop from spinning out of control: the demographic transition and what he calls the ecologic transition. In the ecologic transition, a variety of factors combine to reduce the impact of human activity on the environment (i.e. the u-shaped empirical relationship between *per capita* GDP and environmental quality (Grossman, 1994; Grossman and Krueger, 1995, 1996)). The important issue is the timing of and relationship between these two transitions relative to the degradation of the resource base. Baldwin argues that economic growth is essential for sustainability because it (eventually) brings about the demographic transition. The ecologic transition is necessary to prevent the growth required to bring about the demographic transition from destroying the environment. Thus we should not slow growth in the name of the environment, even with the risk that the ecologic transition may not come in time.

The aim of this paper is to further analyse this complex interaction between demographic and technological factors in a developing economy which is dependent on renewable resources. Specifically, I develop and analyse a two-sector growth model to address three questions. First, what conditions, in terms of investment, demographic, and economic parameters, make a developing economy more or less prone to population overshoot and resource collapse? Second, what is the relative importance of demographic, ecological, and technological adjustments in preventing overshoot and collapse? Finally, what do the answers to these questions suggest about policies affecting economic growth, population dynamics, and the environment in a developing economy?

My analysis departs from the existing work in this area in two important respects. First, I address demographic, economic, and environmental factors together rather than just two of these three. Second, I use dynamical systems techniques to perform a comprehensive analysis of the qualitative behavior of the model. Techniques such as simulation experiments and optimal control address only a very small set of possible growth paths. On the other hand, bifurcation analysis combined with rescaling arguments produce results that are much more general. In fact, the bifurcation techniques I apply make the comprehensive analysis of the model possible.

2. The model

To capture both the positive (Malthusian) and negative (modern growth) type relationships between population growth and output, it is important to model the shifting composition of output from agricultural to manufacturing as growth occurs. Thus, the model is a representative agent, two sector, three factor growth model with a renewable resource base. The two sectors, agriculture and manufacturing, are hereafter labeled as sectors 1 and 2, respectively. The three factors of production are labor, $h(t)$, human-made capital, $k_h(t)$, and natural capital, $k_r(t)$. The human population dynamics are influenced by the *per capita* consumption of goods from each of these sectors.

2.1. Firms

I represent agricultural production with the Cobb–Douglas technology

$$Y_1 = E_1(t) k_r^{\alpha r} L_1^{\alpha_1} K_1^{1-\alpha_1} \tag{1}$$

where Y_1 is output and k_r, K_1, and L_1 represent natural capital, human-made capital, and labor in sector 1, respectively. The function E_1 is a time-dependent measure of efficiency. The technology is constant returns to scale in labor and human-made capital, but exhibits increasing returns overall. The presence of k_r in equation (1) is meant to capture the dependence of agriculture on ecological attributes such as soil structure, fertility, and hydrological processes. Thus k_r should be interpreted not as a stock measure, but as a quality measure. Doubling labor and capital on doubly fertile land can more than double output.

Similarly, manufacturing output, Y_2 is given by

$$Y_2 = E_2(t) L_2^{\alpha_2} K_2^{1-\alpha_2} \tag{2}$$

In this case there is no dependence of Y_2 on natural capital. I do not include this dependence for two reasons: exhaustible resources have been studied in great detail in the context of economic growth (Solow, 1974; Dasgupta and Heal, 1974; Hartwick, 1977), and it does not add significantly to the ideas developed here.

The form of (1) allows for increasing output through increasing cultivation intensity in the sense discussed by Boserup. However, one can argue that not only will population pressure cause increasing cultivation intensity, it will also increase the intensity of the search for new technology. That is, $E_1 = E_1(t, q_1(t), \ldots)$ where $q_1(t)$) is *per capita* output of agricultural goods. For clarity, I carry out the initial analysis with exogenous technical progress ($E_1 = E_1(t)$ then discuss the implications of endogenous technical progress ($E_1 = E_1(t, q_1(t), ...)$).

2.2. Consumers

In the economic growth literature, it has become customary to represent household behavior as an intertemporal optimization problem, i.e. the standard Ramsey model (Barro and Sali-i-Martin, 1995) or an overlapping generations model. A key element in such models is to somehow generate investment supply, either through consumption smoothing, or some life cycle process. The simplest way to generate investment supply is to assume an exogenously set savings rate as in the standard Solow–Swan model.

Again, for clarity, I carry out the initial analysis for the simplest case with the savings rate as an exogenously set parameter. Using the insights from this analysis, I will discuss the effect of endogenizing the savings rate. Given the assumption of a constant savings rate, each identical consumer solves the problem

$$\max U(q_1, q_2) = (q_1)^{c_1}(q_2)^{1-c_1} \tag{3}$$

$$\text{subject to: } P_1 q_1 + P_2 q_2 \leqslant (1-s)M \tag{4}$$

where U is utility, q_1 and q_2 are the per capita consumption rates of agricultural and manufacturing goods, P_1 and P_2 are their respective prices, M is per capita income, s is the savings rate, and $c1$ is the preference for agricultural goods. The familiar solution is (see appendix A.1 for details)

$$q_1 = \frac{c_1 M(1-s)}{P_1} \text{ and } q_2 = \frac{(1-c_1)M(1-s)}{P_2} \tag{5}$$

2.3. General Equilibrium

Let the human population level at time t be denoted by $h(t)$. Each of the h identical consumers owns the same quantity of capital stock. Let labor and human-made capital factor prices be given by w and r respectively. There are five markets in the economy, two factor, two output, and an investment market. Equilibrium occurs when output and factor prices reach equilibrium. Because there are no adjustment costs, and technology is constant returns to scale, firms are indifferent to scale. Firms will make use of all investment in new capital, i.e. investment levels are completely determined by sY where Y is total output. I assume that the manufacturing sector supplies both investment and consumer goods at the same price. The equilibrium level of output in each sector is given by (see appendix A.2)

$$Y_1 = E_1 \beta^{\alpha_1} \gamma^{1-\alpha_1} k_r^{\alpha_r} h^{\alpha_1} k_h^{1-\alpha_1} \tag{6}$$

$$Y_2 = E_2 (1-\beta)^{\alpha_2} (1-\gamma)^{1-\alpha_1} h^{\alpha_1} k_h^{1-\alpha_1} \tag{7}$$

where γ and β are given by

$$\beta = \left(\left(\frac{1}{c_1(1-s)} - 1\right)\frac{\alpha_2}{\alpha_1} + 1\right)^{-1} \tag{8}$$

$$\gamma = \left(\left(\frac{1}{c_1(1-s)} - 1\right)\frac{1-\alpha_2}{1-\alpha_1} + 1\right)^{-1} \tag{9}$$

respectively. Equation (8) reveals that the fraction of labor directed to agriculture is an increasing function of preferences for agricultural goods (c_1) and the productivity of labor in agriculture (α_1). It is a decreasing function of the productivity of labor in manufacturing (α_2). The same statement holds for γ with the word labor replaced with human-made capital.

Investment, I, is computed by substituting the expression for q_2 in expression (5) into (A.2.2) and noting that investment is a proportion of total manufacturing good output (see appendix A.3). This yields

$$I = \frac{sY_2}{1 - c_1(1 - s)} \tag{10}$$

2.4. Dynamic stock equations

In general terms, the three stocks in the model h, k_h, and k_r evolve according to the differential equations

$$\dot{h} = G(h, k_h, k_r)h \tag{11a}$$

$$\dot{k}h = I - \delta k_h \tag{11b}$$

$$\dot{k}_r = F(h, k_h, k_r) - D(h, k_h, k_r) \tag{11c}$$

where G is the *per capita* growth rate of the human population and δ is the depreciation rate of human-made capital. The function F represents the regeneration rate of natural capital while D represents the degradation of natural capital caused by economic activity. Equation (11b) is straightforward. I will now discuss the forms of G, F, and D in turn.

2.4.1. Demographics

There have been several growth models that have endogenized fertility with varying objectives, but often in an attempt to better understand the empirical relationships between economic growth, fertility, and mortality (e.g. Raut, 1990; Barro and Sali-i-Martin, 1995; Galor and Weil, 2000). My purpose, rather, is to understand how these relationships might, via the demographic transition, affect the nature of growth paths.

Cohen (1995) notes that the 'idealized historical pattern' of the demographic transition occurs in four stages: (1) population has both high birth and death rates that are nearly equal leading to slow population growth, (2) death rate falls, birth rate remains high leading to rapid population growth (mortality transition), (3) birth rate falls (fertility transition), and (4) birth and death rates are both low and nearly equal and the population stabilizes at a higher level than at stage (1). Cohen points out that there is often confusion surrounding the interpretation of the 'demographic transition' as the historical process just discussed, or the *hypothetical mechanism* by which the historical process occurs. We are concerned with the latter. In the model, I assume only the essential aspects of this mechanism: income and fertility are negatively correlated as observed in developing economies and mortality is assumed to be negatively correlated with improved nutrition and infrastructure.

To formalize these relationships, I assume that *per capita* manufactured goods output is strongly correlated with overall economic development. Increased manufactured goods output reduces death rates through improved infrastructure, health care, sanitation, education, etc. ('mortality transition'). It reduces birthrates when social change driven by economic growth puts downward pressure on birth rates due to the increased marginal cost of raising children (e.g. see Becker, Murphy, and Tamura, 1990) and changing preferences ('fertility transition'). Note that birth rates are associated with fertility *decisions* as opposed to a physical measure of birth rates. Increased *per capita* output of agricultural goods reduces the death rate through improved nutrition. It puts upward pressure on birth rates through improved overall physical health which causes both earlier onset and higher levels of fertility in females. It may also have an influence on fertility decisions in some societies where wealth is expressed through children (Caldwell, 1976).

To capture these four elements formally, we write

$$G(h, k_h, k_r) = G(q_1(h, k_h, k_r), q_m(h, k_h)) = b(q_1, q_m) - d(q_1, q_m) \qquad (12)$$

where $b(\cdot)$ and $d(\cdot)$ have the obvious interpretations, and q_m is *total per capita* manufacturing output. Here, I assume that

$$b(q_1, q_m) = b_0(1 - e^{-b_1 q_1})e^{-b_2 q_m} \qquad (13)$$

The term $b_0(1 - e^{-b_1 q_1})$ represents increases in birth rates up to a maximum of b_0 as q_1 (i.e. nutrition) increases. The parameter b_1 measures the sensitivity of birth rates to such increases. The term $e^{-b_2 q_m}$ represents downward pressure on birth rates as q_m increases (fertility transition). Again, b_2 measures the sensitivity of birth rates to changes in q_m. Similarly, I assume

$$d(q_1, q_m) = d_0 e^{-q_1(d_1 + d_2 q_m)} \qquad (14)$$

Improved nutrition reduces death rates (i.e. through improved immunity, etc.) via the term $q_1 d_1$ while improved infrastructure reduces death rates via the term $q_1 d_2 q_m$ (mortality transition) where as above, d_1 and d_2 are sensitivity parameters. Note that death rates can only be reduced by improved infrastructure when $q_1 > 0$, i.e. people still starve when they have no food regardless of the level of development of the society in which they live. The parameter d_0 measures the maximum death rate with no nutritional intake.

This simple demographic model captures the four basic linkages between population dynamics and the structure of the economy. The parameters b_i and d_i, $i = 1, 2, 3$ measure the sensitivity of human population dynamics to economic structure. While it is reasonable to link death rates and the state of the system deterministically (i.e. agents do not, in general, choose their death rates), it is less realistic to link birth rates and the state of the system in this way. However, I am interested in capturing only the aggregate interaction between fertility and the state of the system for which equation (13) is sufficient. I will discuss a more realistic model for fertility choices in section 3.5.

2.4.2. The renewable resource system

Interacting biological communities and processes (natural capital) provide inputs to agricultural production such as clean irrigation water, hydrological balance, soil nutrients, and soil structure maintenance. Agricultural practices necessarily disturb these communities and processes reducing the services they contribute with, in some cases, drastic consequences (Quiggin, 1988; Lefkoff and Gorelick, 1990). To maintain productivity, the goods and services supplied by these forms of natural capital must be replaced by substitutes such as fertilizer and complex water management systems generated using human-made capital.

It is impossible to accurately model the ecological processes described above. They can exhibit highly non-linear behavior causing the natural capital base to change rapidly and unpredictably (Carpenter, Ludwig, and Brock, 1999). For our purposes, all that need be captured is the fact that on time scales relevant to economic systems the populations described above, if undisturbed, will grow up to maximum levels limited by the physical environment. A simple way to capture this behavior is with a logistic growth function; a very common way to model density dependent regeneration of a bioresource (Clark, 1990; Anderies, 1998; Brander and Taylor, 1998). Natural capital regenerates itself logistically and is degraded by agricultural production, thus

$$F(h, k_h, k_r) = n_r k_r (1 - k_r) \tag{15a}$$

$$D(h, k_h, k_r) = \eta Y_1 \tag{15b}$$

By choice of units, k_r lies in the interval [0, 1]. The parameter n_r is the aggregate intrinsic regeneration rate of the renewable resource base, and η measures the impact of farming on the natural capital base.

3. Analysis of the model

To answer the questions posed in the introduction, we must catalogue the model behavior for all possible economic, demographic, and ecological conditions. Each of these conditions is related to a particular parameter combination in the model. Thus our task is to divide the entire parameter space into regions in which the model exhibits different qualitative behavior. Using ideas from the mathematical theory of dynamical systems (Kuznetsov, 1995) combined with a powerful technique of computer-based bifurcation analysis (Doedel, 1981), I catalogue the qualitative behavior of the model over the *entire parameter space*. This analysis reveals fundamental principles about the nature of growth paths in a developing economy which are more general than those that can be obtained with simulation experiments or optimal control. Hundreds, perhaps thousands, of simulation experiments would be required to map out the qualitative model behavior as a function of parameters. This makes it an impractical approach, providing, at best, an incomplete picture of the possible qualitative behavior of the model. Optimal control techniques necessarily focus on very specific trajectories rather than on the general topological properties of growth paths which are our focus here.

There are two main classes of development paths possible in the model. In one case, any reasonable initial condition with high biophysical capital and low population will evolve to a stable steady state (perhaps through a series of damped oscillations). In the other case, no reasonable initial condition with high biophysical capital and low population can evolve to a steady state. Rather, it will converge to a limit cycle. The existence of the limit cycle acts as a topological barrier between any initial condition and a long-run sustainable growth path. From a development perspective, oscillations are undesirable because they represent periods of increasing *per capita* consumption followed by periods of decreasing *per capita* consumption. The intergenerational equity problems associated with such trajectories are obvious. We wish to explore how changing parameters (i.e. economic, demographic, and ecological conditions) cause the model dynamics to shift between these two behaviors.

These two qualitative behaviors are separated by a Hopf bifurcation which occurs when the steady state changes from being locally stable to unstable as a parameter is varied. A periodic orbit then develops around the now unstable steady state. The bifurcation analysis amounts to starting at a known stable equilibrium of the system and tracking its stability as a parameter is varied in very small steps. By locating points where the stability of the fixed point changes, we can detect local bifurcations and use these to divide the parameter space as mentioned above. This powerful tool for analyzing dynamical systems is freely available. Interested readers should visit http://www.math.pitt.edu/~phase for more details and download information. For more details on the application of the method for ecological models see (van Coller, 1997).

3.1. Critical points

The model exhibits three critical points (see appendix A.4 for details): $(h, k_h, k_r) = (0, 0, 0)$, $(h, k_h, k_r) = (0, 0, 1)$, and $(h, k_h, k_r) = (h^*, k_h^*, k_r^*)$ such that $h^* > 0$, $k_h^* > 0$ and $0 < k_r^* < 1$. The point (0, 0, 0) is unstable, meaning that, if not exploited, natural capital will increase to its maximum. The stability of the point (0, 0, 1) depends on parameter choices. Stability of this point means that the ecological economic system represented by this parameter set is not 'viable'. That is, the resource base is not sufficiently productive to support a human population with a given technology. If this point is unstable, the system is 'viable'. Given a viable parameter set, it follows that there exists an interior equilibrium point (see appendix A.4). Table 1 summarizes a set of viable parameters used in the model analysis. Note that several parameters are scale factors and are set by choice of units.

3.2. Investment and the demographic transition

In this section, analysis is conducted to support three propositions: (1) if investment dynamics are fast relative to the regeneration rate of the resource base, investment is a fundamentally destabilizing force making the system more prone to overshoot and collapse; (2) if the feedback between manufactured goods consumption and birth rates b_2 is sufficiently strong, the fertility transition can prevent overshoot and collapse; and (3) the higher the savings rate s and the stronger the feedback between manufactured goods consumption and the death rate d_2, the higher b_2 must be to prevent overshoot and collapse.

Throughout this and subsequent analysis, we are focusing on time scales one might expect for a society to develop from an agricultural society to an industrial society once the industrialization process gets under way. The

Table 1. *Parameter values used in the model for the numerical bifurcation analysis*

Parameter	*Definition*	*Value*
Economic parameters		
α_1	Labor productivity in the resource sector	varies
α_2	Labor productivity in the manufacturing sector	varies
α_r	Resource productivity	0.75
c_1	Resource good preference	0.3
δ	Depreciation rate of capital	0.05
s	Savings rate	varies
$E_i(t)$	Efficiency factor in sector i	varies
Ecological parameters		
η	Effect of resource good production on resource base	0.1
n_r	Intrinsic rate of increase of resource base	0.1
Demographic parameters		
b_0	Maximum birth rate	0.1
d_0	Maximum death rate	0.2
b_1	Sensitivity of birth rate to resource good intake	1
b_2	Sensitivity of birth rate to manufactured good intake	varies
d_1	Sensitivity of death rate to resource good intake	5
d_2	Sensitivity of death rate to manufactured good intake	varies

demographic parameters were chosen to reproduce growth rates that roughly agree with observed rates over the last century (Cohen, 1995). Depending on parameter choices, this produces dynamics that occur on time scales of 100–400 years (see figure 2). For this baseline case, I assume no technological progress and set $E_i = 1$ for $i = 1, 2$. The parameters are set as shown in table 1 along with $\alpha_1 = 0.7$ and $\alpha_2 = 0.3$.

We first illustrate the long-term behavior of the model as a function of s with $b2 = d2 = 0$. Figure 1A is a bifurcation diagram that plots the long-run

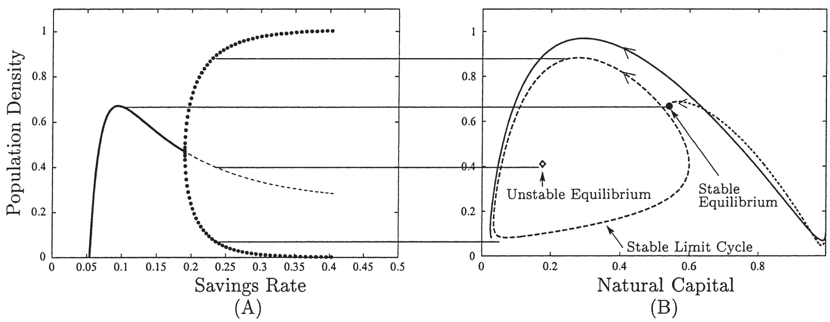

Figure 1. *Graph (A) shows the bifurcation diagram for the model with the basic parameter set from table 1 as s is varied. Graph (B) shows the trajectories in phase space that eventually converge to the long-run configurations (i.e. stable equilibrium or limit cycle) shown in (A) for a low savings rate (s = 0.09, dashed line) and a high savings rate (s = 0.23, solid line). The horizontal lines show the correspondence between (A) and (B). For s = 0.09, the model converges and for s = 0.23, the model overshoots and collapses.*

equilibrium population for different levels of investment. The solid line for lower values of s indicates that these equilibria are stable. The dashed portion of the curve for higher values s indicates that the equilibria are unstable and the system will never approach them. Rather, for values of s above the Hopf bifurcation point near where the curve of heavy circles intersects the line, the system will approach a stable limit cycle, whose amplitude is shown by the heavy solid circles emanating from the curve of fixed points. Figure 1B shows the transient behavior and long-run equilibria in $h-k_r$ phase space corresponding to points in figure 1A. The horizontal lines connect points in the bifurcation diagram with their counterparts in phase space. The reader should bear in mind that the phase space in figure 1B shows only two of the three dimensions (i.e. is a two-dimensional projection) of the model. For clarity, the third dimension k_h has been suppressed. Both of the two other possible projections, the $h-k_h$ and k_h-k_h phase space, have the same qualitative structure. Figure 2A shows the time paths for human population for the two cases corresponding to figure 1B, and figure 2B shows the respective birth and death rates.

These diagrams show how increased savings rates destabilize the model. Investment increases productive capacity allowing the population to overexploit the resource base (Clark, Clarke, and Munro, 1979) and grow beyond the long-run carrying capacity of the resource base. The population must subsequently (perhaps painfully) adjust downwards. This 'overshoot and collapse' behavior is generated by differing time scales in the non-linear model. If the time scale on which productive capacity grows is faster (slower) than that on which the resource base regenerates itself, the more (less) prone the system will be to overshoot and collapse.

The importance of *relative* time scales in determining model dynamics can be made explicit by rescaling time. Define $\tau \equiv n_r t$. The new variable, τ, measures time relative to the natural time scale implicit in n_r. The model can then be rewritten with respect to this new timescale using the chain rule, i.e.

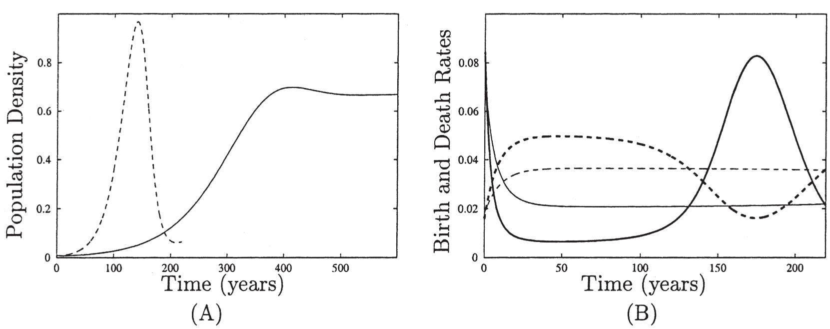

Figure 2. *Graph (A) shows the population density over time for s = 0.09 (solid line) and s = 0.23 (dashed line). Graph (B) shows the corresponding birth and death rates. The solid (dashed) lines show death (birth) rates over time for s = 0.09 (light lines) and s = 0.23 (heavy lines).*

$$\frac{dh}{d\tau} = \frac{dh}{dt}\frac{dt}{d\tau} = \frac{dh}{dt}\frac{1}{n_r} \tag{16}$$

Performing this operation on each of the equations in the model yields

$$\frac{dh}{d\tau} = (\hat{b}(q_1, q_m) - \hat{d}(q_1, q_m))h \tag{17}$$

$$\frac{dk_h}{d\tau} = \frac{\hat{s}Y_2}{1 - c_1(1 - s)} - \hat{\delta}k_h \tag{18}$$

$$\frac{dk_r}{d\tau} = k_r(1 - k_r) - \hat{\eta}Y_1 \tag{19}$$

where ˆ indicates division by n_r, i.e. $\hat{s} = \frac{s}{n_r}$. The topological properties of this model are equivalent to the original model. For a given set of parameters, this model will exhibit a Hopf bifurcation when $\hat{s}$ exceeds a certain level. Here the interpretation is more revealing – as $\hat{s}$ increases, the *relative* time scale in the economic system becomes faster than in the natural system. For the parameter set used in the example, a Hopf bifurcation would occur when $\hat{s} = \frac{s}{n_r} \approx \frac{0.2}{0.1} = 2$, i.e. when the economic time scale is roughly twice the natural time scale. What is important is not the absolute savings rate, but the savings rate relative to the intrinsic replacement rate of the resource base.

By setting $b_2 > 0$ we now examine the potential for the fertility transition to prevent overshoot and collapse. The main effect of increasing the rate of investment is to increase the difference between the birth and death rates as illustrated in figure 2B. The heavy lines show the birth (dashed) and death (solid) for $s = 0.23$ while the light lines show the same for $s = 0.09$. In the first case, large increases in productive capacity cause a large difference in birth and death rates leading to rapid population growth (around a maximum of 4 per cent per year). This leads to resource degradation and eventually to very high death rates peaking around year 170 at 8 per cent annually. In the latter case, the difference between birth and death rates is relatively small resulting in a growth rate of less than 1.7 per cent per annum. This results in a slower degradation of the resource base as the birth and death rates approach their equilibrium value of around 3.1 per cent. In both cases, the equilibrium 'throughput' is high. A more desirable situation would be one with low 'throughput', achieved by decreasing birth rates via increases in q_2. Note that after the growth process begins, the birth and death rates (and thus *per capita* consumption) stabilize after roughly 30–50 years. It then takes approximately 100 years after this point to degrade the environment (for $s = 0.23$). It is in this first five decade period of growth in *per capita* consumption that the fertility transition must take place.

Figure 3 summarizes the effect the fertility transition has on growth paths. The graph on the right shows the two regions in $s - b_2$ parameter space that exhibit qualitatively different behavior. The curve that separates the two regions is generated by locating the Hopf bifurcation point for each combination of s and b_2. The three graphs on the left show the popu-

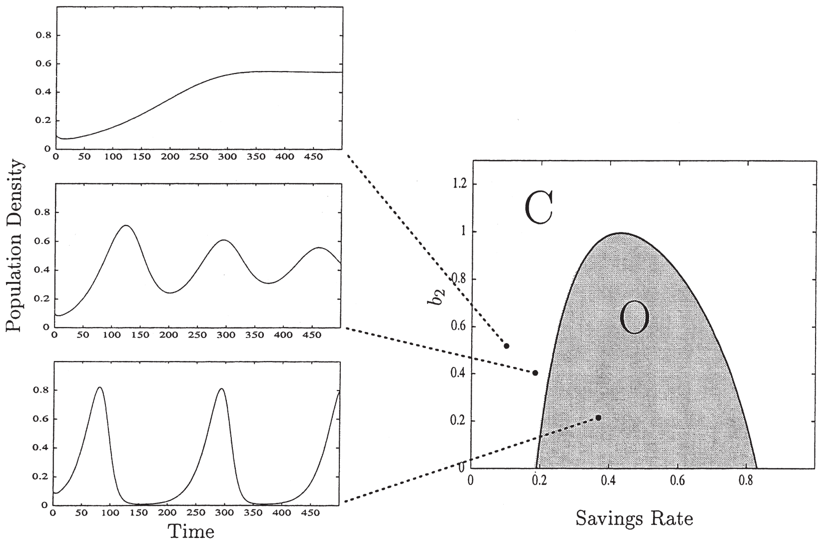

Figure 3. *Depicted in the graph on the right is the separation of* $s - b_2$ *parameter space into two regions. Points on the curve represent combinations of* s *and* b_2 *for which a Hopf bifurcation occurs, i.e. a qualitative change in model behavior occurs at these points. The three graphs on the right show human population density as a function of time for parameter values located by the dotted lines. For all* $s - b_2$ *parameter combinations in region* 'C'*, the model converges to long run equilibrium. For all* $s - b_2$ *parameter combinations in region* 'O'*, the model oscillates indefinitely.*

lation trajectories over time corresponding to the points in parameter space to which they are connected with a dotted line. The cases shown in the top two graphs both eventually converge to a long-run equilibrium as do all of the cases with parameter combinations in the region marked 'C' (convergence). The bottom graph never converges; it oscillates indefinitely as do all cases for parameter combinations in the shaded region marked 'O' (overshoot and collapse). Notice that even though the model can experience some overshoot and collapse as it converges for parameter combinations in region 'C', they are much less severe than the overshoot and collapse for parameter combinations in region 'O'. It is interesting to note that recent archaeological work suggests that human societies that have exhibited 'O' type behavior are very common (Tainter, 1988; Redman, 1999).

The implication of this analysis is that the greater the savings rate s, the greater b_2 (the strength of the fertility transition) must be to prevent overshoot and collapse. In fact, if $b_2 > 1$ the system does not undergo a Hopf bifurcation. The inverted u-shape of the 'O' region is a result of the fact that with extremely high savings rates the population cannot afford to feed itself and thus grows very slowly (not a very realistic scenario). Thus increasing b_2 allows faster growth without overshoot and enables the system to reach stage four of the demographic transition with low birth and death rates. For example, with $b_2 = 1$, and $s = 0.2$, the equilibrium birth (and death rate) is approximately 1.1 per cent versus 3.1 per cent with $b_2 = 0$ and $s = 0.09$.

The parameter b_2 could have many different physical interpretations and I offer only one possibility here. Different distributions of income across a particular economy can give rise to the same average *per capita* income. The homotheticity of the Cobb–Douglas utility function makes income distribution irrelevant when computing aggregate demand (demand is a linear function of income), but the same may not be true of birth rates. Suppose, as with preferences, each agent has the same response for birth rate to consumption. If this function is non-linear, then the aggregate birth rate will depend on income distribution.

For example, if economic development is not even, some individuals might enjoy certain benefits that reduce mortality without experiencing other aspects of the development process that might suppress birth rates. In this case the response of the birth rate to consumption levels would be weak (modeled by a low value of b_2). If, however, the benefits of economic growth are distributed evenly, birth rates would fall off more quickly as consumption increased because more individuals in the population would reduce births for the same level of average *per capita* intake (high b_2). Whatever the physical interpretation of b_2, the key point is that it is largely socially determined.

I conclude this section with an analysis of the effect the other demographic parameter, d_2, has on the model. Recall that d_2 measures the sensitivity of death rates to *per capita* manufactured goods output. With both b_2 and d_2 non-zero, economic growth sets up a tug-of-war between the fertility transition (stage 3 of the demographic transition), and the mortality transition (stage 2 of the demographic transition). Figure 4A shows how the bifurcation boundary shifts upward and to the left as d_2 is increased from 0 to 1. When d_2 is 0, the 'O' region is indicated by dark gray shading. When d_2 is 1, the 'O' region is enlarged by the area shown with light gray shading. The larger the effect growth has on reducing death rates the greater must be its effect on reducing birth rates to avoid overshoot and collapse dynamics. This common-sense result highlights the double-edged sword nature of economic development. Figure 4B shows the explicit dependence of the minimum strength of the fertility transition, b_2, required to avoid overshoot and collapse dynamics on the strength of the mortality transition, d_2. The dotted (solid) line shows this relationship for $s = 0.15$ ($s = 0.2$). The arrows indicate two possible trajectories that b_2 and d_2 could take over time if endogenized. If d_2 increases due to improved medical technology (e.g. $b_2 = b_2(k_h)$), either b_2 must increase proportionately (upward-sloping arrow) or the system will enter the undesirable 'O' region (horizontal arrow). The larger the savings rate, the sooner the system enters the 'O' region.

3.3. Economic structure

In this section, it is shown that the more capital intensive the agricultural sector (lower α_1) and the overall economy, the more difficult it is to avoid

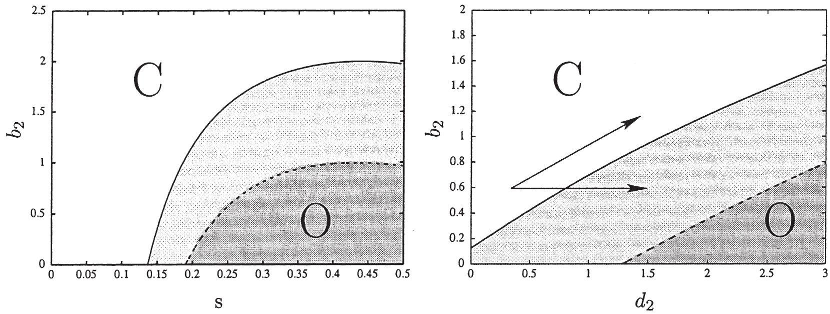

Figure 4. *In graph (A), The dashed line is a portion of the curve that separates regions 'O' and 'C' shown in figure 3 for $d_2 = 0$. The grey region below the dashed line marked 'O' has the same interpretation as in figure 3. The solid curve shows the division between the 'O' and 'C' regions for $d_2 = 1$. As d_2 increases, the size of the overshoot and collapse region expands by an amount shown by the light gray shading. Graph (B) illustrates the trade-off between b_2 and d_2 (the influence of manufactured good consumption on birth and death rates, respectively). Points on the lines represent parameter combinations for which a Hopf bifurcation occurs for $s = 0.15$ (dot-dashed line) and $s = 0.2$ (solid line). These lines divide parameter space into regions with qualitatively different behavior.*

overshoot and collapse. From (8) it is easy to see that the labor fraction in the *i*th sector decreases with decreasing α_i. Increasing α_2 relative to α_1 increases the proportion of available labor devoted to manufacturing. The parameter choice used in the analysis thus far ($\alpha_1 = 0.3$, and $\alpha_2 = 0.7$) is roughly consistent with a modern society where relatively less labor is employed in agriculture than in manufacturing.

Figure 5 summarizes the relationship between capital intensity and the fertility transition. The savings rate is set at 0.2 and all other parameters are as shown in table 1 with E_i both still set at 1, and $d_2 = 0$. The heavy curve shows the boundary between 'O' and 'C' type dynamics in $b_2 - \alpha_1$ parameter space with α_2 set at 0.7. The lighter curve shows the same for α_2 set at 0.5.

For a given value of b_2, as the capital intensity in agriculture increases (α_1 decreases) the system moves from the 'C' region to the 'O' region. The lower b_2, the higher the value of α_1 below which the system will exhibit overshoot and collapse behavior. The lighter curve illustrates the effect of increased capital intensity in the overall economy. Two points are worth noting. First, when b_2 is 0, the labor intensity below which the system becomes unstable is higher (approximately 0.46 when $\alpha_2 = 0.5$ versus 0.35 when $\alpha_2 = 0.7$). Second, for larger values of b_2, the difference between the boundaries becomes less marked. This is due to the fact that when the capital intensity increases in the manufacturing sector, *per capita* output of manufactured goods (q_2) increases. This, in turn, increases the strength of the downward pressure on birth rates for a given value of b_2. Figure 5 shows that in societies where the bulk of the labor force is engaged in

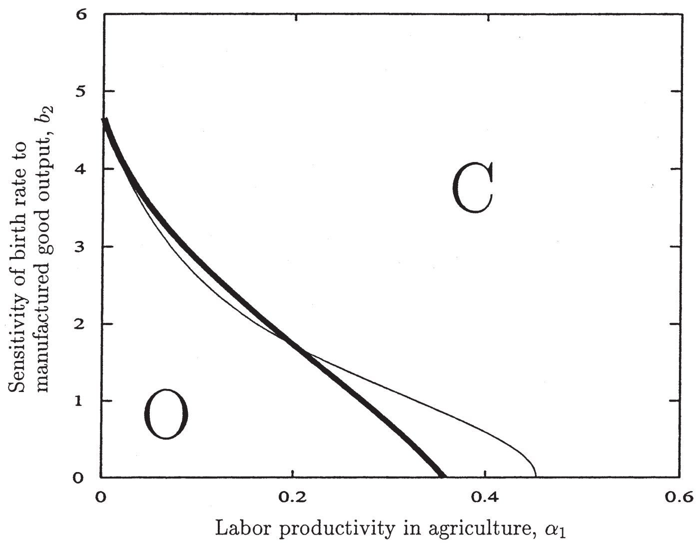

Figure 5. *Division of $b_2 - \alpha_1$ parameter space into regions of qualitatively different behavior. In region 'C' the model converges. In region 'O' the model overshoots and collapses. The heavy (light) line shows the division between the two regions when $\alpha_2 = 0.7$ ($\alpha_2 = 0.5$).*

manufacturing the more important the 'fertility transition' becomes in avoiding degradation of the natural resource base.

3.4. Technological change

How does dynamically evolving technology effect the results presented thus far? This is an extremely difficult question because of the fundamentally speculative nature of technological change. In the case of endogenous growth theory, the question is how innovation might cause growth (Grossman and Helpman, 1991; Lucas, 1988; Romer, 1986). These models need only assume that innovation is taking place somewhere most of the time. For example, overall productivity growth in an economy might be the result of a small segment of the economy (e.g. computer hardware and the US economy in the decade from 1990–2000 – Gordon, 1999). It is irrelevant that the productivity gains are exceedingly narrow, they still generate growth. In our model, however, it is important that *certain types of innovation* occur at the *right time*. In this section, first the bifurcation analysis is extended to show that exogenous technological change that enhances productivity makes the system more prone to overshoot and collapse. Second, a scaling argument is used to show that technological change that reduces the impact of economic activity on the environment (clean technology) cannot, in general, prevent overshoot and collapse.

Including exogenous technical change amounts to making E_1 and E_2 increasing functions of time. Including technological change that reduces the environmental impact of agriculture can be modeled by making η a decreasing function of time. To explore the role of technological progress via increases in E_1 and E_2, the demographic parameters are fixed and E_i are varied. By varying them we can determine how the qualitative behavior of the model will change as E_i change over time. These quantities affect the model dynamics in three basic ways: E_1 and E_2 affect human population dynamics by increasing q_i, E_1 increases the per capita impact on the environment via equation (15), and E_2 increases the rate of growth of the capital stock (and thus the long-run capital–labor ratio). Figure 6 summarizes the effects of changes in E_i on the model. In order to compare with previous results, the parameters are set as shown in table 1 with $\alpha_1 = 0.3$, $\alpha_2 = 0.7$, $s = 0.1$, and $b_2 = d_2 = 1$.

The solid arrows indicate possible time trajectories that exogenous or endogenous technological progress could take. Notice that with E_i both fixed at 1 as previously, the model will converge to a long-run equilibrium and will not overshoot and collapse. As is clear from the figure, technological advance that increases productivity is destabilizing. The more even the advance of $E_1(t)$ and $E_2(t)$ (i.e. the 45° line) the sooner the model becomes unstable. The more uneven the advance of $E_1(t)$ and $E_2(t)$, the longer it takes for the model to become unstable.

The destabilization occurs through the increasing impact of agricultural production on the natural resource base given by

$$\eta Y_1 = \eta E_1 \beta^{\alpha_1} \gamma^{1-\alpha_1} k_r^{\alpha_r} h^{\alpha_1} k_h^{1-\alpha_1} \tag{20}$$

This occurs directly through E_1 and indirectly through E_2 via increases in k_h. To prevent the inevitable destabilization of the model, this impact must eventually stop increasing. This can be achieved several ways. The most

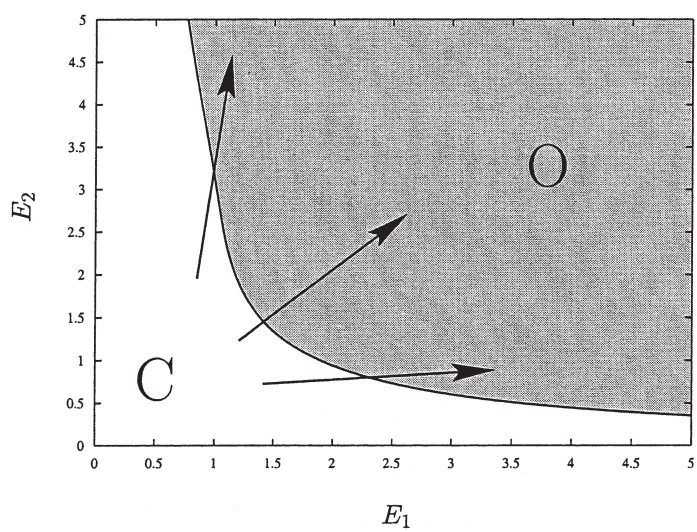

Figure 6. *Division of $E_2 - E_1$ parameter space into convergence ('C') and overshoot and collapse ('O') regions. The arrows indicate possible paths E_2 and E_1 might take as they evolve over time.*

direct way is for technological progress to cease before E_1 and E_2 move into region 'O'. Another way is through technological progress that causes $\eta(t)$ to *decrease* over time at a rate fast enough to compensate for increases in $E_i(t)$. Thus as production becomes more efficient, its impact on the environment must decrease over time. The model highlights the fact that outside a simple one sector growth model the role technological progress may play in achieving sustainable outcomes is not obvious. A balancing act between different types of progress is required.

Now consider exogenous technological progress that reduces the impact of production on the resource base. Suppose this progress causes $\eta(t)$ to decrease over time at a constant rate r_η. To see its effect, we make the following change of variables: $\hat{h} = \eta h$, and $\hat{k}_h = \eta k_h$. Now $\hat{h}$ and $\hat{k}_h$ are measured in terms of their impact on the environment. Under this transform, $Y_1(h, k_h, k_r) = Y_1(\hat{h}, \hat{k}_h, k_r)/\eta$, and $Y_2(h, k_h) = Y_2(\hat{h}, \hat{k}_h)/\eta$. The change of variables does not affect *per-capita* quantities, so the model can be rewritten as follows

$$\dot{\hat{h}} = \eta\dot{h} + h\dot{\eta} = \eta(b(q_1, q_m) - d(q_1, q_m))\frac{\hat{h}}{\eta} + \hat{h}\frac{\dot{\eta}}{\eta} = (b(q_1, q_m) - d(q_1, q_m) - r_\eta)\hat{h} \tag{21}$$

$$\dot{\hat{k}}_h = \eta\dot{k}_h + k_h\dot{\eta} = \eta\left[\frac{s\frac{Y_2(\hat{h},\hat{k}_h)}{\eta}}{1 - c_1(1-s)} - \delta\frac{\hat{k}_h}{\eta}\right] + \hat{k}_h\frac{\dot{\eta}}{\eta} = \frac{sY_2(\hat{h}, \hat{k}_h)}{1 - c_1(1-s)} - (\delta + r_\eta)\hat{k}_h \tag{22}$$

$$\dot{k}_r = n_r k_r(1 - k_r) - \eta\,\frac{Y_1(\hat{h}, \hat{k}_h, k_r)}{\eta} = n_r k_r(1 - k_r) - Y_1(\hat{h}, \hat{k}_h, k_r) \tag{23}$$

The additional constant r_η is the only difference between equations (21) and (22) and the original model. Equation (23) appears different but, since $Y_1(\hat{h}, \hat{k}_h, k_r) = \eta Y_1(h, k_h, k_r)$, is not.

Note in (21) that the new term r_η can reduce the growth rate of $\hat{h}$. Bifurcation analysis like that carried out for s and b_2 in section 3.2 shows that a similar relationship holds for s and r_η. As with b_2, the larger s, the larger r_η must be to prevent overshoot and collapse. On the surface, this result supports Baldwin's (1994) claim that improving technology that reduces the impact of economic activity on the environment, the ecologic transition, can prevent population overshoot and resource collapse. However, a closer inspection reveals that this is not the case.

The role of r_η, a rate of change, is fundamentally different from the role of b_2, the strength of a feedback. Increasing and maintaining r_η at a positive level requires that technological progress reduce the impact of economic activity on the resource base indefinitely. This, in turn, requires that $\eta(t)$ tend toward 0 over time. The values of r_η required to prevent overshoot and collapse in the model can be 5–10 per cent. On the time scale of 50–100 years we are considering, this requires $\eta(t)$ to decrease by one to three orders of magnitude. Because $h(t) = \hat{h}(t)/\eta$ and $k_h(t) = \hat{k}_h(t)/\eta$ such a reduction in $\eta(t)$ implies a one to three order of magnitude increase in h

and k_h. Thus, a constant r_η requires the impact of economic activity on the environment to approach 0 in five generations and implies perpetual population growth ($\dot{h}\backslash h = r_\eta$). This is implausible and suggests that either (1) r_n would eventually fall to 0 due to thermodynamic constraints, (2) a constraint not included in the model checks growth (e.g. global warming), or (3) a feedback from the state variables (global information about the over all scale of the system) influences either fertility decisions or mortality and checks population growth.

In the first case, when $r_n \rightarrow 0$ (and $\eta(t) \rightarrow \eta^*$), the rescaled model becomes equivalent to the original model and thus has the same behavior. The model will either approach an equilibrium value ($\hat{h}^*$, $\hat{k}_h^*$) or will overshoot and collapse (approach a limit cycle in the variables $\hat{h}$ and $\hat{k}_h$). In this case, technical progress that reduces η simply increases the scale of the long-run sustainable population if overshoot is avoided, or causes the model to overshoot and collapse on a grander scale if not. The key point is that whether overshoot and collapse is avoided in this case depends on other parameters in the model, not technological change. In the second case, decreasing $\eta(t)$ merely shifts pressure from one resource to another. Thus, in order for decreasing η to prevent overshoot and collapse, one is forced to assume that a similar process is occurring for all potential resource constraints. Finally, the last case suggests either that people constrain fertility based on global information (national population) rather than local information (cost of raising children) or some other process such as disease or war check population growth.

This analysis shows that without a global feedback from the overall scale of the system to individual's fertility decisions, cleaner technology (i.e. the ecologic transition) either is irrelevant or causes more problems than it solves. If fertility decisions are based solely on *per capita* quantities, the ecologic transition merely increases the scale at which the dynamic tension between economic growth and resource degradation is played out. Only changes in social parameters (e.g. increasing b_2 or decreasing s relative to n_r) can prevent overshoot and collapse.

3.5. Endogenizing technological change and investment patterns

In reality, s, η, and E_i are not parameters. They evolve depending on feedbacks from the state of the system, e.g. $\dot{s} = C(h, k_h, K_r)$. In this final section, I address the effect of endogenizing investment and technological change on the qualitative behavior of the model. It is not necessary to model these feedbacks explicitly. This complicates the mathematics without sufficient compensation in new insights. The results of the bifurcation analysis and the rescaling argument above are sufficient to understand the effects of these endogenous feedbacks. Increasing E_i (endogenously or exogenously) is destabilizing as depicted in figure 6. This is so because increasing E_i simply increases the ability of the population to exploit the resource base. Decreasing η (endogenously or exogenously) does not change the basic topology of the system, it simply magnifies the total scale of the human enterprise the resource base can support – either sustainably or in an overshoot and collapse mode.

This leaves us with the feedbacks between the state of the system, fertility, and the savings rate. These are often captured by assuming optimizing households either in a variant of the Ramsey model or an overlapping generations model. Such models raise many difficult questions. What are households optimizing? Are they simply smoothing consumption or making quality quantity trade-offs in their fertility choices? Is environmental quality an argument in household utility functions? What information is available for decision making?

To be realistic, households must set at least investment and fertility rates. They would also set what proportion of their investment goes to increasing k_h and to reducing η. Already we have three state variables and three control variables. This exploding model complexity makes optimal control analysis intractable and is part of the reason that the three part problem of population dynamics, economic growth, and environmental change is so challenging to meaningfully address in an optimizing framework. It also highlights the utility of the dynamical systems approach presented herein.

For example, consider a standard optimal growth model with fertility

$$\max_{b,q_1,q_2} U = \int_0^{\infty} \frac{e^{-\rho t}}{1-\theta} \left\{ [q_1^{c_1} q_2^{c_2} h^{c_3} d^{-c_4}]^{1-\theta} - 1 \right\} dt \tag{24a}$$

subject to

$$\dot{h} = (b - d(q_1, q_2))h \tag{24b}$$

$$\dot{\tilde{k}}_h = \frac{w - q_1 P_1 - q_2 P_2 - c_b b}{P_2} + \tilde{k}_h \left(\frac{r}{P_2} - \delta - b + d(q_1, q_2) \right) \tag{24c}$$

$$\dot{k}_r = n_r k_r (1 - k_r) - h q_1 \tag{24d}$$

$$h = L_1 + L_2 \tag{24e}$$

$$k_h = K_1 + K_2 \tag{24f}$$

where I have assumed the usual constant elasticity of intertemporal substitution utility structure with θ, c_i, $i = 1, \ldots, 4$ all non-negative. Households derive utility based on family size h (since all households are identical, h is a proxy for household size), and *per capita* consumption, q_1 and q_2. High death rates generate disutility, and thus c_4 is preceded by a negative sign. Households choose their fertility, b, and their consumption levels, q_1, and q_2. Equation (24c) is the household wage constraint, which determines the evolution of the capital–labor ratio, $\tilde{k}_h$. Finally, c_b is the cost of raising children.

Including (24d) as a constraint generates a global feedback of the kind discussed in section 3.4. This feedback links k_r to fertility decisions and prevents households from over saving and from choosing too high fertility rates. Such choices will generate overshoot and collapse trajectories which will be penalized by the death rate term in the utility function. This outcome, however, rests on many assumptions including omniscient households with perfect foresight. Policy based on such analysis would apply in only very special circumstances. Policy based on the dynamical systems approach, however, relies on far fewer assumptions and would apply in a wider range of circumstances.

4. Concluding remarks

I have presented an analysis of the interaction of demographic and technological factors in a developing economy that depends on renewable resources. Using dynamical systems techniques, several results were obtained concerning the effect of investment and demographic patterns, economic structure, and technological change on the qualitative nature of development paths. Taken together, these results point to two important conclusions:

1. If the rate of renewable resource generation is slow relative to the rate of economic growth, population overshoot and resource degradation become more likely.
2. Demographic factors are relatively more important in preventing population overshoot and collapse of the resource base than technological factors. This basic result is summarized in figures 4 and 6 along with the rescaling argument in section 3.4. Increasing b_2 can prevent overshoot and collapse while technology, in the absence of a direct feedback between global state variables and individual fertility decisions, either has no effect, or favors overshoot and collapse. In this case, economic growth and technological change actually increase, rather than decrease, the importance and urgency of the role demographic factors play in preventing renewable resource degradation.

The second conclusion highlights the importance of the use of a dynamical systems approach for research in this area. Comparing the analysis presented here to the traditional optimal control framework highlights the narrow view of the development process provided by the latter. The dynamical systems analysis reveals that a feedback between individual fertility decisions and global state variables is essential for economic growth and technological change to play a significant role in avoiding resource degradation. As discussed in section 3.5, the optimal control framework tends to build such feedbacks in. Since it is questionable whether such feedbacks exist in developing economies, conclusions drawn from optimal control analysis may not be that useful in policy design. Future work combining dynamical systems and optimal control techniques will form a broader picture of the development process and possibly help improve policy design.

Finally, the model suggests that policy should focus on social processes that govern the way agents invest and reproduce. Without a strong and well-understood feedback between the overall scale of human activity and demographic processes, policies that encourage increased economic growth and clean technology should not be relied upon to mitigate resource degradation while waiting for the demographic transition to occur. Developed economies in a position to influence the growth of less-developed economies face the challenge of actively addressing the social and cultural processes that drive investment and reproductive behavior while at the same time encouraging growth on a time scale that will not generate overshoot and collapse type dynamics.

A Appendix

A.1 Consumer optimization

To derive the demand functions in (5) from the constrained optimization problem given in (4) we define the Lagrangian

$$\mathcal{L} \equiv U(q_1, q_2) - \lambda[P_1 q_1 + P_2 q_2 - M(1 - s)]. \tag{A.1.1}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial q_1} = c_1 q_1^{c_1 - 1} q_2^{1 - c_1} - \lambda P_1 = \frac{c_1 U}{q_1} - \lambda P_1 = 0 \tag{A.1.2}$$

$$\frac{\partial \mathcal{L}}{\partial q_2} = (1 - c_1) q_1^{c_1} q_2^{-c_1} - \lambda P_2 = \frac{(1 - c_1)U}{q_2} - \lambda P_2 = 0 \tag{A.1.3}$$

Adding (A.1.2) and (A.1.3) yields

$$\frac{U}{\lambda} = P_1 q_1 + P_2 q_2 = M(1 - s) \tag{A.1.4}$$

Solving (A.1.2) and (A.1.3) for q_1 and q_2, respectively, and substituting the right-hand side of (A.1.4) for $\frac{U}{\lambda}$ yields (5).

A.2 General equilibrium

Assume firms rent capital from individuals. Savings is used to purchase investment goods from the manufacturing sector which are added to the capital stock. The price of investment goods is the same as for consumer goods. The total demand for agricultural and manufactured goods is then

$$Y_1^D = h q_1 \tag{A.2.1}$$

$$Y_2^D = h q_2 + \frac{sM}{P_2} \tag{A.2.2}$$

where the superscript indicates 'demanded'. Perfectly competitive firms solve:

$$\max_{L_i, K_i} \pi_i(L_i, K_i) = P_i Y_i - w L_i - r K_i \tag{A.2.3}$$

where $\pi_i(L_i, K_i)$ is profit in sector $i = 1, 2$. The first-order conditions are

$$\frac{\partial \pi}{\partial L_i} = \frac{\alpha_i P_i Y_i}{L_i} - w = 0 \tag{A.2.4}$$

$$\frac{\partial \pi}{\partial K_i} = \frac{(1 - \alpha_i) P_i Y_i}{K_i} - w = 0 \tag{A.2.5}$$

Adding (A.2.4) and (A.2.5) for $i = 1, 2$ yields the supply equations

$$P_1 Y_1^S = w L_1 + r K_1 \tag{A.2.6a}$$

$$P_2 Y_2^S = w L_2 + r K_2 \tag{A.2.6b}$$

Output markets clear when

$$P1Y_1^D = P_1Y_1^S \Leftrightarrow c_1hM(1-s) = wL_1 + rK_1 \tag{A.2.7}$$

$$P_2Y_2^D = P_2Y_2^S \Leftrightarrow (1-c_1)hM(1-s) + sM = wL_2 + rK_2 \tag{A.2.8}$$

Since technology is constant returns to scale, there is nothing in the equations so far to determine the scale of economic output. I thus assume full capital utilization, i.e.

$$L_1 + L_2 = h \text{ and } K_1 + K_2 = k_h \tag{A.2.9}$$

Factor markets clear when labor and capital rental rates, w and r adjust to reflect the relative scarcity of total available labor, h, and capital, k_h. Equations (A.2.4), (A.2.5), (A.2.7), and (A.2.9) can be used to calculate K_1 as follows: Note that

$$hM = wh + rk_h \tag{A.2.10}$$

by definition. Given r and w, (A.2.4) and (A.2.5) imply that firms will choose capital–labor ratios that satisfy

$$\frac{K_i}{L_i} = \frac{w(1-\alpha_i)}{r\alpha_i}. \tag{A.2.11}$$

This combined with (A.2.10) allows (A.2.7) to be written as

$$c_1(1-s)\left[\frac{rK_1}{1-\alpha_1} + \frac{r(k_h - K_1)}{1-\alpha_2}\right] = \frac{r\alpha_1K_1}{1-\alpha_1} + rK_1 \tag{A.2.12}$$

Solving this for K_1 and plugging this result into the expression on the right in (A.2.9) yields

$$K_1 = \gamma k_h \text{ and } K_2 = (1-\gamma)k_h \tag{A.2.13}$$

where γ is given by

$$\gamma = \left(\left(\frac{1}{c_1(1-s)} - 1\right)\frac{1-\alpha_2}{1-\alpha_1} + 1\right)^{-1} \tag{A.2.14}$$

Given this capital allocation, the labor demand in each sector as dictated by (A.2.11) is then

$$L_1 = \frac{r\gamma k_h\alpha_1}{w(1-\alpha_1)} \text{ and } L_1 = \frac{r(1-\gamma)k_h\alpha_2}{w(1-\alpha_2)} \tag{A.2.15}$$

To close the system, choose r as the numeraire and set it equal to 1. It is now easy to see that labor demand is determined by total available capital stock, and wage rates. If total labor demand is higher (lower) than the total available, h, there will be upward (downward) pressure on the wage rate. The wage rate will adjust until the expression on the left in (A.2.9) is satisfied, factor markets clear, and the whole system converges to the general equilibrium. Substituting the expressions for labor demand in (A.2.15) into (A.2.9) and solving for the equilibrium wage rate yields

$$w = \frac{k_h}{h}\left[\frac{\gamma\alpha_1}{(1-\alpha_1)} + \frac{(1-\gamma)\alpha_2}{(1-\alpha_2)}\right] \tag{A.2.16}$$

Substituting this expression for the wage rate into the labor demand equations in (A.2.15) yields the equilibrium labor allocations given by

$$L_1 = \beta h \text{ and } L_2 = (1 - \beta)h \tag{A.2.17}$$

with β given by

$$\beta = \left(\left(\frac{1}{c_1(1-s)} - 1\right)\frac{\alpha_2}{\alpha_1} + 1\right)^{-1} \tag{A.2.18}$$

Finally, combining equations (1), (2), (A.2.13), and (A.2.17) yields (6) and (7).

A.3 Investment

Note that as with most models of growth and the environment, total investment is completely supply-side driven. This is due to the fact that there are no adjustment costs in the model. Recall from (A.2.8) that the total demand for manufactured goods is comprised of total consumption goods, hq_2 and investment goods, $\frac{shM}{P_2}$. At equilibrium

$$P_2Y_2 = h(1 - c_1)M(1 - s) + hsM \tag{A.3.1}$$

which when rearranged gives

$$\frac{hM}{P_2} = \frac{Y_2}{1 - c_1(1 - s)} \tag{A.3.2}$$

Thus investment is given by

$$I = \frac{shM}{P_2} = \frac{sY_2}{1 - c_1(1 - s)} \tag{A.3.3}$$

A.4 Equilibria and local stability analysis

The model

$$\dot{h} = (b(q_1, q_m) - d(q_1, q_m))h \tag{A.4.1}$$

$$\dot{k}_h = \frac{sY_2}{1 - c_1(1 - s)} - \delta k_h \tag{A.4.2}$$

$$\dot{k}_r = n_r k_r(1 - k_r) - \eta Y_1 \tag{A.4.3}$$

has three equilibria

$$(h, k_h, k_r) = (0, 0, 0) \tag{A.4.4}$$

$$(h, k_h, k_r) = (0, 0, 1) \tag{A.4.5}$$

$$(h, k_h, k_r) = (h^*, k_h^*, k_r^*),\ h^* > 0,\ k_h^* > 0,\ k_r^* \in (0, 1) \tag{A.4.6}$$

The first two equilibria follow immediately from the model equations. We will establish conditions for the existence of the third (non-trivial) equilibrium below.

Standard methods for checking the stability of the first two points are not applicable because the Jacobian does not exist when h and k_h are 0. It is easy to show that the point (0, 0, 0) is unstable by considering a perturbation of the form (0, 0, ϵ). For such a perturbation, both $\dot{h}$ and $\dot{k}_h$ are 0 while $\dot{k}_r > 0$. Thus the perturbation will tend to grow, and the system will move away from (0, 0, 0). This implies (0, 0, 0) is unstable.

To check the stability of the second point, consider a perturbation of the form $(\epsilon_1, \epsilon_2, 0)$.

Define

$$A \equiv \left[\frac{sE_2(1-\beta)^{\alpha_2}(1-\gamma)^{1-\alpha_2}}{\delta(1-c_1(1-s))}\right]^{\frac{1}{\alpha_2}} \tag{A.4.7}$$

Equation (A.4.2) implies

$$\frac{k_h}{h} \leqslant A \Leftrightarrow \dot{k}_h \geqslant 0 \tag{A.4.8}$$

Thus perturbations such that $\epsilon_1 > \frac{\epsilon_2}{A}$ will tend to grow in the k_h dimension until $\frac{k_h}{h} \to A$. Similarly, perturbations such that $\epsilon_1 < \frac{\epsilon_2}{A}$ will tend to decay in the k_h dimension until $\frac{k_h}{h} \to A$. The question of the stability of this equilibrium (and the existence of the third equilibrium) then hinges on whether this perturbation will grow in the h dimension as well, or will decay back to 0.

Fix all the parameters in the model except b_0. Rewriting equations (6) and (7) in *per capita* terms yields

$$q_1(x, k_r) = E_1\beta^{\alpha_1}\gamma^{1-\alpha_1}k_r^{\alpha_r}x^{1-\alpha_1} \tag{A.4.9}$$

$$q_2(x) = E_2(1-\beta)^{\alpha_2}(1-\gamma)^{1-\alpha_1}x^{1-\alpha_1} \tag{A.4.10}$$

where x is the capital–labor ratio, $\frac{k_h}{h}$. Choose b_0 such that

$$b_0 > \frac{d_0 e^{-q_1(A,1)(d_1+d_2q_2(A))}}{e^{-b_2q_2(A)}\left(1-e^{-b_1q_1(A,1)}\right)} \tag{A.4.11}$$

For parameter sets satisfying (A.4.11), at the equilibrium capital–labor ratio A and with $k_r = 1, \dot{h} > 0$ so the population will grow. Increasing the population will tend to decrease x below A, all else being equal. When this occurs, equation (A.4.8) implies that k_h will grow. Thus a perturbation of the form above will tend to grow in both h and k_h dimensions away from (0,0,1) – i.e. (0,0,1) is unstable. If, however, the inequality in condition (A.4.11) is reversed, at the equilibrium capital–labor ratio A and with $k_r = 1, \dot{h} < 0$ so the population will decay. As h decreases, x will tend to increase above A, all else being equal. When this occurs, equation (A.4.8) implies that k_h will decay. Thus a perturbation of the form above will tend to decay in both h and k_h dimensions back to (0,0,1) – i.e. (0,0,1) is stable.

Condition (A.4.11) describes a parameter set for a 'viable' system. Parameter combinations that satisfy (A.4.11) yield a model system in which the resource base can support a human and human-made capital population. Otherwise, the resource base, given technology and preferences, cannot meet the needs of the human population. Our final task is, given a 'viable' system, to establish the existence and uniqueness of the equilibrium point (h^*, k_h^*, k_r^*).

If this equilibrium exists, $\frac{k_h^*}{h^*} = A$, and (A.4.1) implies that

$$\xi(k_r^*) \equiv b_0 e^{-b_2 q_2(A)} \left(1 - e^{-b_1 q_1(A,k_r^*)}\right) - d_0 e^{-q_1(A,k_r^*)\,(d_1+d_2 q_2(A))} = 0 \quad \text{(A.4.12)}$$

Note that if (A.4.11) is satisfied, $\xi(1) > 0$ and it is easy to check that $\xi(0) < 0$. The continuity of ξ and the Intermediate Value Theorem imply that $\exists\, k_r^* \in (0, 1)$ such that $\xi(k_r^*) = 0$. Next note that

$$\xi'(k_r^*) = b_0 e^{-b_2 q_2(A)}\, e^{-b_1 q_1(A,k_r^*)}\, b_1 \frac{\partial q_1}{\partial k_r} + d_0 e^{-q_1(A,k_r^*)(d_1+d_2 q_2(A))}\, (d_1 + d_2 q_2(A)) \frac{\partial q_1}{\partial k_r} > 0 \quad \text{(A.4.13)}$$

The Mean Value Theorem along with (A.4.13) implies that $\exists$ at most one k_r^* such that $\xi(k_r^*) = 0$. This combined with the existence statement above implies that there exists a unique k_r^* such that $\xi(k_r^*) = 0$. Then

$$h^* = \frac{k_r^* n_r (1 - k_r^*)}{\eta E_1 \beta^{\alpha_1} \gamma^{1-\alpha_1} (k_r^*)^{\alpha_r} A^{1-\alpha_1}} \quad \text{(A.4.14)}$$

and

$$k_h^* = A h^* \quad \text{(A.4.15)}$$

Thus, for parameter sets satisfying (A.4.11), we have established the existence and uniqueness of an equilibrium point (h^*, k_h^*, k_r^*) with $h^* > 0$, $k_h^* > 0$, $k_r^* \in (0, 1)$. In general, there is no closed-form solution for equation (A.4.12). Thus the analysis of the equilibrium point (h^*, k_h^*, k_r^*) is carried out numerically as outlined in the text.

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